Coupled flow and mechanics in a fractured porous medium

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We present a mixed dimensional model for a fractured poro-elasic medium including contact mechanics. The fracture is a lower dimensional surface embedded in a bulk poro-elastic matrix. The flow equation on the fracture is a Darcy type model that follows the cubic law for permeability. The bulk poro-elasticity is governed by fully dynamic Biot equations. The resulting model is a mixed dimensional type where the fracture flow on a surface is coupled to a bulk flow and geomechanics model. This contact conditions are in the form of normal compliance and Coulomb friction. The resulting variational problem consists of a linear partial differential equation coupled to a nonlinear variational inequality. We show the well-posedness of the resulting model and provide an iterative scheme that is contractive. Furthermore, we propose and analyze a fully discrete numerical scheme for the Biot model with contact conditions using conformal finite elements in space and the implicit Euler method in time. The Biot equations with the contact conditions forms the key result for the fractured setting. Existence and uniqueness of the discrete solution is established, and stability and a priori error estimates are derived. A numerical experiment is performed in which numerical error estimates are computed and compared to the theoretical results.

This is a joint work with Maarten V de Hoop, Marius Nevland, Inga Berre, Jakub Wiktor Both, Eirik Keilegavlen.

References

[1] M. V. de Hoop and K. Kumar, *Coupling of flow, contact mechanics and friction, generating waves in a fractured porous medium*, J. Math. Anal. Appl. 560(1), 130416 (2026)

[2] M. Nevland, K. Kumar, I. Berre, J. W. Both, and E. Keilegavlen, *Numerical analysis of the Biot equations coupled to frictional contact mechanics*, arXiv preprint arXiv:2607.02030 (2026)

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